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# MSW effect with flavor changing interactions and the atmospheric neutrino problem

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## Abstract

We consider flavor changing effective neutrino interactions in the context of massive neutrinos in the issue of atmospheric neutrinos. Assuming as usual that this is an indication of the oscillation of muon neutrinos into tau neutrinos we show that there is a set of parameters which is consistent with the MSW resonance condition for the typical Earth density and atmospheric neutrino energies. In particular we show that even if the vacuum mixing angle vanishes it is still possible to have a resonance which is compatible with the atmospheric neutrino data. We also briefly consider the case of the solar neutrino problem.

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It is a well accepted fact that the solar [1] and atmospheric neutrino data [2] strongly suggest that neutrinos are massive particles [3]. Other solutions like pure flavor changing neutrino interactions [4] or neutrino magnetic moments [5,6] are still possible for the solar neutrino problem, at least with the present data. Although the more favoured solution is the vacuum oscillation one, it is still possible that two or more of these mechanisms are in action simultaneously. For instance, the MSW [7,8] effect together with effective flavor changing neutrino interactions was proposed in Ref. [9]. This is an interesting possibility since almost all extensions of the electroweak standard model implies new interactions for the neutrinos.

On the other hand, several years ago Bethe [10] re-derived the resonance effect on neutrinos propagating through a medium [7,8] based on a different view: his approach consists on considering the addition of a term $2EV$ upon the neutrino mass square due to the $W$-interactions of massive neutrinos propagating in a medium; $E$ is the neutrino energy and $V$ is the potential energy due to the matter effect. Here we will consider a similar effect when new interactions with matter (electrons or quarks) do exist and also if the neutrinos have arbitrary masses and for the sake of simplicity we will treat only two generations.

In this situation, the neutrino symmetry eigenstates $\nu_\alpha$ and $\nu_\beta$ are related to the neutrino mass eigenstates $\nu_1$ and $\nu_2$ as follows

$$|\nu_\alpha\rangle = |\nu_1\rangle\cos\theta + |\nu_2\rangle\sin\theta, \quad |\nu_\beta\rangle = -|\nu_1\rangle\sin\theta + |\nu_2\rangle\cos\theta \tag{1}$$

and $\theta < 45^o$; the masses of the states $|\nu_{1,2}\rangle$ are $m_{1,2}$ respectively and as we said before, we are assuming that they are arbitrary parameters. The square of the mass matrix in the $|\nu_{\alpha,\beta}\rangle$ flavor basis is given by

$$\mathcal{M}^2 = \frac{1}{2}(m_1^2+m_2^2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}(m_2^2-m_1^2)\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}. \tag{2}$$

Assuming effective interactions of the form $(G_{\alpha f}G_{f\beta}/M_X^2)(\bar\nu_\alpha\nu_\beta)(\bar f f)$, $\alpha,\beta = e,\mu,\tau$; and $M_X$ is the mass of the interchanged particle and $f$ denotes a charged lepton or quark (we have omitted Lorentz indices) implies a potential energy that is given by (here we will consider only interactions with the electrons i.e., $f = e$):

$$V_{\alpha\beta}^{NI} = \frac{x}{M_W^2}\,G_{\alpha e}G_{e\beta}N_e, \quad \alpha,\beta = e,\mu,\tau; \tag{3}$$

with $x = M_W^2/M_X^2$; $N_e$ is the number of electrons per unit volume. Constraints on $x$ and $G_{\alpha\beta}$ coming from several decays are model dependent so they will not be considered here at all. However, see Ref. [4].

In fact, our considerations are model independent but neutrinos must be massive particles with renormalizable (arbitrary) masses. This is a necessary condition in order to have neutrino masses that do not depend on the parameters of the flavor changing interactions $G_{\alpha e}$. For instance, in the context of the $R$-parity broken MSSM [11] it means that one has to add right-handed neutrinos and its respective sneutrinos $\tilde\nu_R$. In the so called 3-3-1 model [12] neutrinos can have arbitrary Majorana masses if a neutral component of the sextet gains a non vanishing vacuum expectation value [13], or we can also add three right-handed neutrinos (and Dirac neutrinos), or both possibilities.

The momentum of a neutrino is related to its energy $E$ by

$$k^2 + m^2 = (E - V)^2 \approx E^2 - 2VE, \tag{4}$$

and considering the atmospheric neutrino issue i.e., the muon and tau neutrino we see that $V_{\alpha\beta}$ is equivalent to an addition to $m^2$ given by

$$m^2_{\alpha\beta} = 2V_{\alpha\beta}E = 2\sqrt{2}\left(\frac{x}{\sqrt{2}}\, G_{\alpha e}G_{e\beta} + G_F M_W^2(1 - 4s_W^2)\,\delta_{\alpha\beta}\right)\frac{Y_e}{M_W^2 m_n}\,\rho E \equiv A_{\alpha\beta} \tag{5}$$

where $\alpha, \beta = \mu, \tau$; $m_n$ is the mass of the nucleon and $Y_e$ the number of electrons per nucleon in the matter, usually $Y_e = 1/2$ and we have introduced the $Z^0$ contribution as well. Taking the usual values for the parameters [14] we have:

$$A_{\alpha\beta} = 1.0 \times 10^{-3}\left(\frac{x}{\sqrt{2}}\, G_{\alpha e}G_{e\beta} + 0.006\,\delta_{\alpha\beta}\right)\rho E, \tag{6}$$

where $\rho$ is in grams per cubic centimeter, $E$ is in GeV, and $A_{\alpha\beta}$ is in $\mathrm{eV}^2$.

Thus, in a dense medium the original vacuum mass square matrix is modified by the $A_{\alpha\beta}$ term and it reads:

$$\tilde{\mathcal{M}}^2 = \frac{1}{2}\left(m_1^2 + m_2^2 + A_{\mu\mu} + A_{\tau\tau}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} A_{\mu\mu} - A_{\tau\tau} - \Delta\cos 2\theta & 2A_{\mu\tau} + \Delta\sin 2\theta \\ 2A_{\mu\tau} + \Delta\sin 2\theta & -A_{\mu\mu} + A_{\tau\tau} + \Delta\cos 2\theta \end{pmatrix}. \tag{7}$$

where $\Delta = m_2^2 - m_1^2$ and we will assume $m_2^2 > m_1^2$. The eigenvalues are

$$\tilde{m}^2_{1,2} = \frac{1}{2}\left(m_1^2 + m_2^2 + A_{\mu\mu} + A_{\tau\tau}\right) \pm \frac{1}{2}\left[(-A_{\mu\mu} + A_{\tau\tau} + \Delta\cos 2\theta)^2 + |2A_{\mu\tau} + \Delta\sin 2\theta|^2\right]^{1/2}, \tag{8}$$

and we see that the resonance condition is given by:

$$A_{\mu\mu} - A_{\tau\tau} = \Delta\cos 2\theta. \tag{9}$$

In Fig. 1 we show the neutrino energy $E$ as a function of the Earth density at the resonance given by Eq. (9) for a given set of parameters, in particular with vanishing vacuum mixing angle. In Fig. 2 we show the two eigenvalues $\tilde{m}^2_{1,2}$ as a function of $E$ and not as a function of $\rho$ and we see that a conversion is possible for the typical Earth density and neutrino energies also with $\cos 2\theta = 1$. The occurrence of this resonance with $\theta = 0$ does not occur in the context of the standard model. Both figures only illustrate that typical values for the Earth density and atmospheric neutrino energies are compatible with the resonance picture discussed above.

In fact, we can also consider the survival probability in a medium which is given by

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\tilde{\theta}\sin^2\left(1.27\,\tilde{\Delta}\,\frac{L}{E}\right), \tag{10}$$

where

$$\sin^2 2\tilde{\theta} = \frac{(2A_{\mu\tau} + \Delta \sin 2\theta)^2}{(\Delta \cos 2\theta - A_{\mu\mu} + A_{\tau\tau})^2 + (2A_{\mu\tau} + \Delta \sin 2\theta)^2}, \quad \text{and} \quad \tilde{\Delta} = \tilde{m}_2^2 - \tilde{m}_1^2, \tag{11}$$

with $\tilde{\Delta}$ in $\mathrm{eV}^2$, $L$ in km and $E \approx \tilde{E} = |\vec{p}|$ in GeV; and we have already used $\tilde{E}_2 - \tilde{E}_1 \approx \tilde{\Delta}/2|\vec{p}|$.

Notice that we can explain the atmospheric neutrino data since at the resonance $\sin^2 2\tilde{\theta}_r = 1$ and if $\tilde{\Delta}_r = 2.2 \times 10^{-3}\,\mathrm{eV}^2$ (the subscript $r$ denotes the value at the resonance), which corresponds to the best fit of atmospheric neutrinos [2,15] leaving the vacuum parameters $\theta$ and $\Delta$ arbitrary. We have

$$\tilde{\Delta}^2 = (\tilde{m}_2^2 - \tilde{m}_1^2)^2 = (-A_{\mu\mu} + A_{\tau\tau} + \Delta \cos 2\theta)^2 + |2A_{\mu\tau} + \Delta \sin 2\theta|^2, \tag{12}$$

and we see that, even at the resonance, if $\theta = 0$, we can still have that $\tilde{\Delta}_r = 2|A_{\mu\tau,r}| = 2.2 \times 10^{-3}\mathrm{eV}^2$, which is accomplished, for instance, for neutrinos with $E \approx 2$ GeV by taken $G_{\mu e} \approx G_{\tau e} \approx 1$, $\rho = 2\,\mathrm{g/cm}^3$. This also occurs if $\Delta = 0$ (neutrinos mass degenerated or massless neutrinos) which also implies $A_{\mu\mu} = A_{\tau\tau}$ at the resonance. This case corresponds to the pure flavor changing induced neutrino oscillations as in Ref. [16] but this situation may have problems with the up-going upward muon [15] as has been pointed out in Ref. [17].

Likewise, we can consider the solar neutrino problem. In this case we have to change labels, $\mu \to e$, $\tau \to \mu$, in the expressions above and also to add the contribution of the $W$ boson. Then

$$m_{\alpha\beta}^2 = 2V_{\alpha\beta}E = 2\sqrt{2}\left(\frac{x}{\sqrt{2}}\, G_{\alpha e}G_{e\beta} + G_F M_W^2 \left[\delta_{\alpha e}\delta_{\beta e} + (1 - 4s_W^2)\delta_{\alpha\beta}\right]\right) \frac{Y_e}{M_W^2 m_n}\, \rho E \equiv A_{\alpha\beta}, \tag{13}$$

for $\alpha, \beta = e, \mu$. Putting values we have

$$A_{\alpha\beta} = 1.0 \times 10^{-3} \left(\frac{x}{\sqrt{2}}\, G_{\alpha e}G_{e\beta} + 0.075\, \delta_{\alpha e}\delta_{\beta e} + 0.006\, \delta_{\alpha\beta}\right) \rho E, \tag{14}$$

with the same units as in Eq. (6).

The eigenvalues are

$$\tilde{m}_{1,2}^2 = \frac{1}{2}\left(m_1^2 + m_2^2 + A_{ee} + A_{\mu\mu}\right) \pm \frac{1}{2}\left[(-A_{ee} + A_{\mu\mu} + \Delta \cos 2\theta)^2 + |2A_{e\mu} + \Delta \sin 2\theta|^2\right]^{1/2}. \tag{15}$$

$\Delta$ has the same definition as before, however the masses $m_{1,2}$ may be different from that of the muon and tau neutrino case. In this case we have a similar resonance condition than that in Eq. (9). In Fig. 3 we show neutrino energies as a function of typical Sun density at the resonance for a given set of parameters and using $\cos 2\theta = 1$ again. The occurrence of that resonance at $\theta = 0$ does not happens if we consider the electron neutrino and the muon neutrino as in the Bethe's paper [10]. The latter case is got here if $A_{ee} = A$, $A_{\mu\mu} = A_{e\mu} = 0$ and $\theta \neq 0$.

Since in the present context the atmospheric neutrino data is almost independent of the value of vacuum mass square difference, it means that we could be able to explain solar [1], atmospheric [2,15] and LSND [18] neutrino data with three active neutrinos and without

introducing a sterile neutrino. This will be the case if, in a truly three generation case, the $\Delta_{12}$ which dominates the $\nu_\mu \to \nu_\tau$ oscillation also dominates the $\nu_e \to \nu_\tau$ transition; while $\Delta_{13}$ is the one used for fitting the LSND results.

We have worked in the two-neutrino scheme because most of the data are presented in this situation. However a general three-neutrino scheme must be worked elsewhere; also more realistic fittings by considering the matter density profile of the Earth and Sun eventually have to be done [19].

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FIGURES

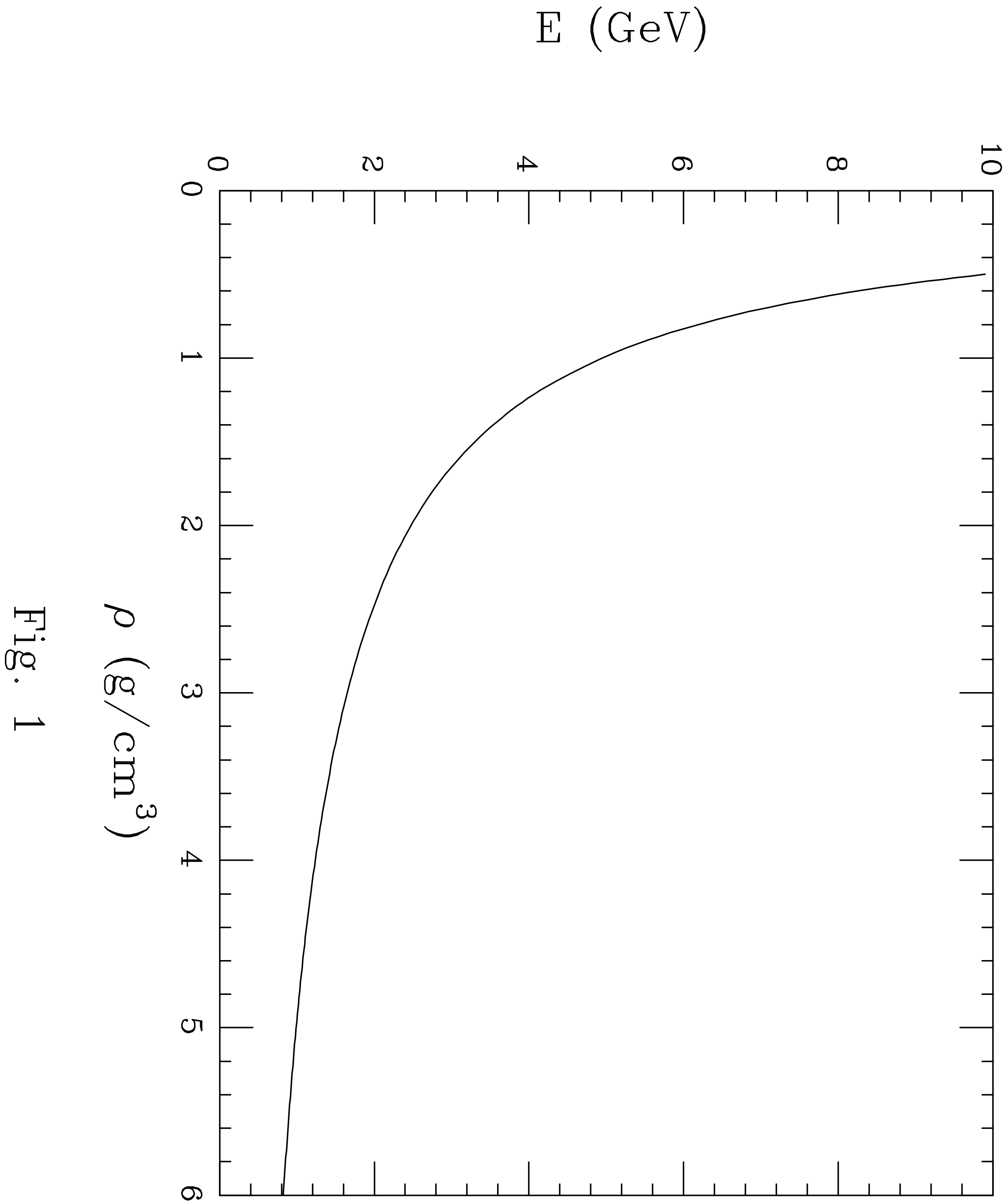


FIG. 1. The energy of the neutrinos satisfying the resonance condition as a function of the Earth density for $G_{\mu e} = 0.9$, $G_{\tau e} = 0.25$, $\Delta = 2.2 \times 10^{-3}\mathrm{eV}^2$ and $\cos 2\theta = 1$.

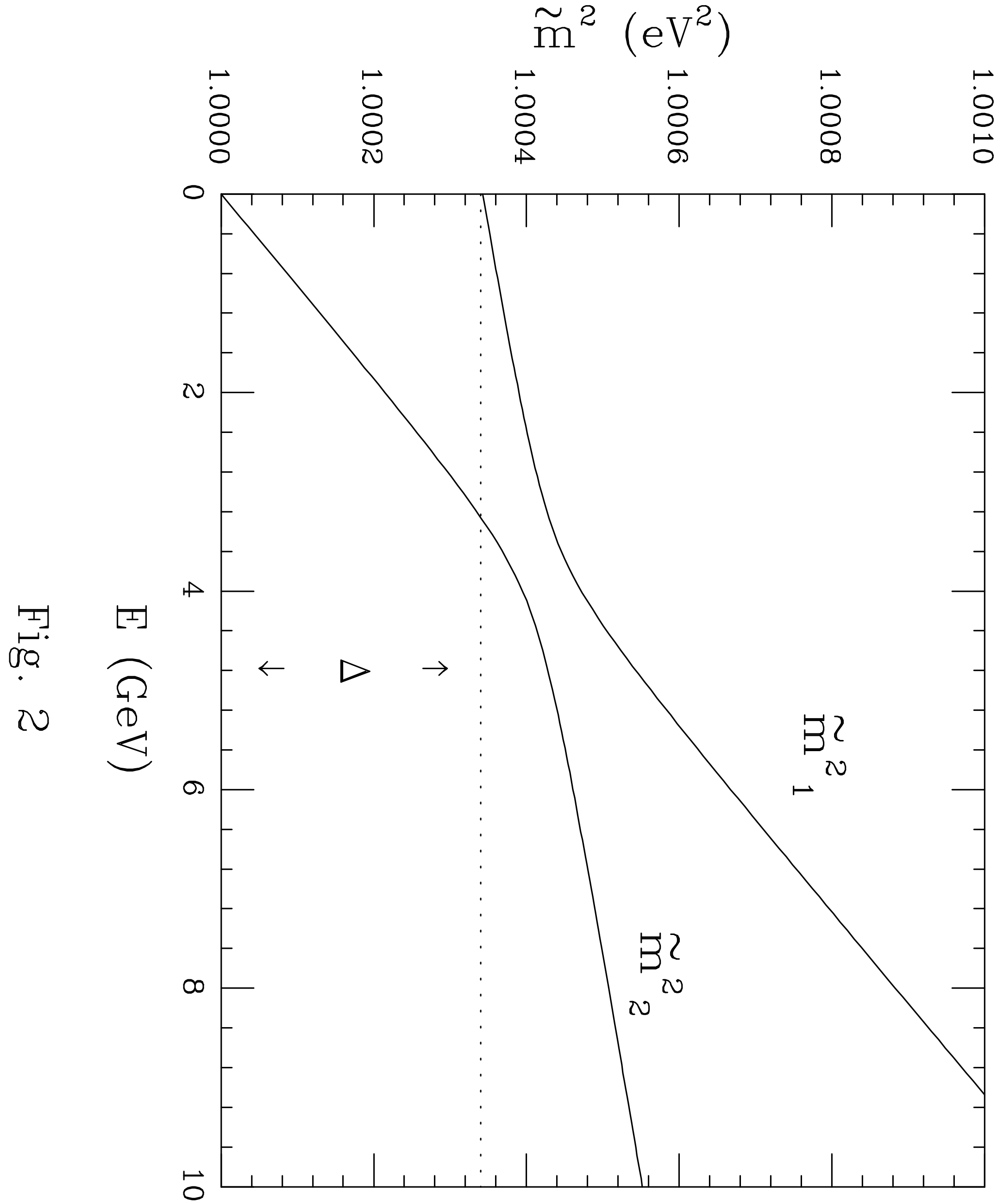


FIG. 2. The quantities $\tilde{m}^2_{1,2}$ as functions of the neutrino energy for $\rho = 2$ g/cm$^3$, $m_1^2 = 1$ eV$^2$, $m_2^2 = m_1^2 + \Delta$ with $\Delta = 3.4 \times 10^{-3}$ eV$^2$ and $\cos 2\theta = 1$.

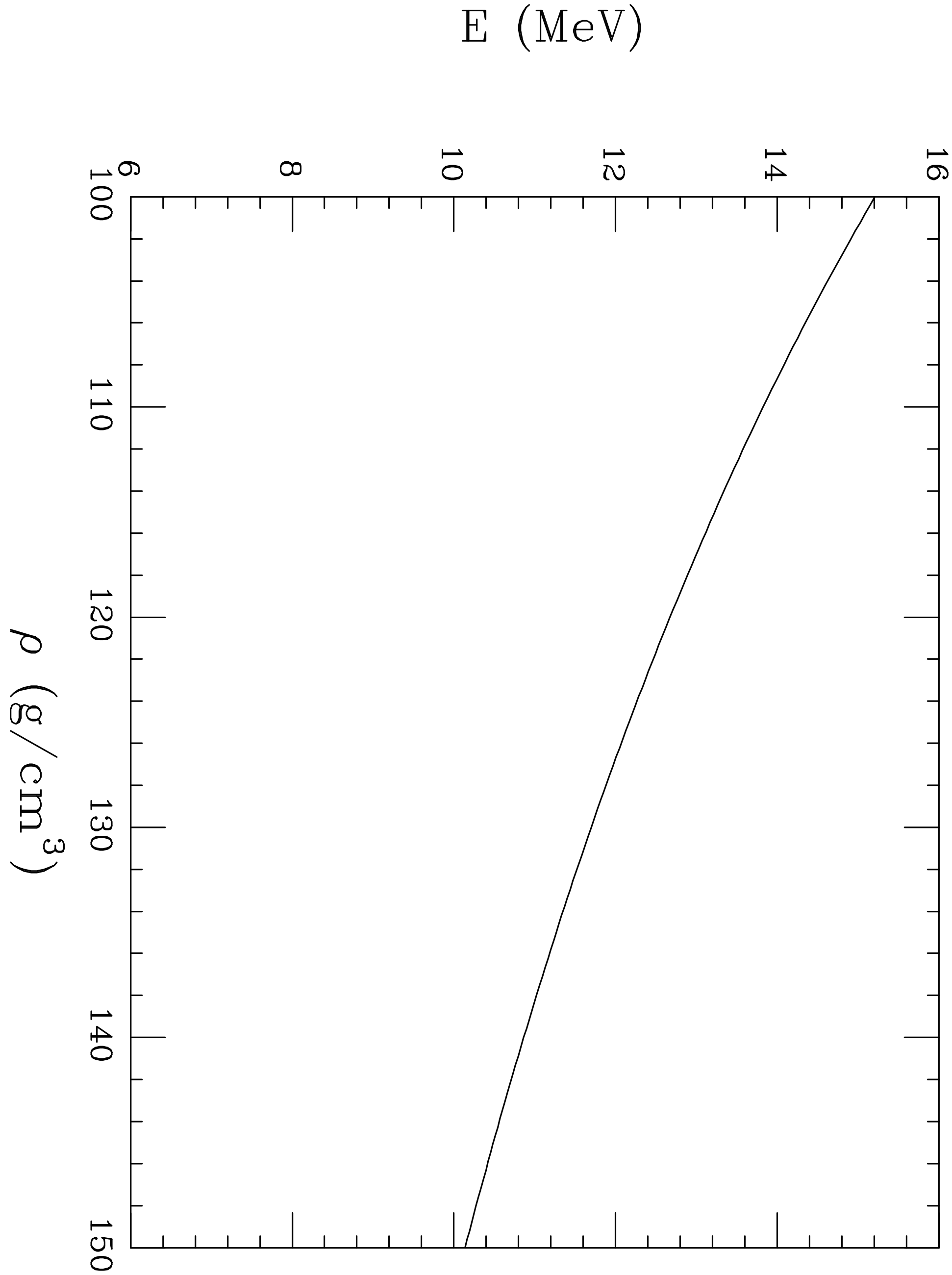


Fig. 3

FIG. 3. The same as in Fig. 1 with the Sun density and $G_{ee} = 1.0$, $G_{\mu e} = 0.9$, $\Delta = 2.0 \times 10^{-4}\mathrm{eV}^2$ and $\cos 2\theta = 1$.